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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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EXPANSIONS OF RANDOM PROCESSES AND THEIR NONCOMMUTATIVE CONVERSIONS.

Ya. P. Dragan, (L'vov).

Analysis is given of representation of correlation functions of random processes and the processes themselves in the form of linear combinations of specific determined functions of time with random coefficients: general spectral, canonical, orthogonal. It is shown that in contrast to the case of the commutative converters, analyzed in the terms of theory of C*-algebras and their Gelfand isomorphism, in the case of noncommutative converters it is necessary to use operators of transformation of coordinates, composition of Volterra and Δ-composition. Is introduced the concept of T-representability, which generalizes the concept of harmonizability. As examples are examined the harmonic modulated oscillation/vibration, the periodically correlated processes, sampling theorems. No tables. Bibliog. 14.

The problems of the linear theory of signal are solved within the framework of the covariance theory random process, which are represented in the form of the linear combination of the determined functions of time with the random coefficients.

In this article different forms of such expansions and their change under action of linear transducers are analyzed, and is also examined one of the possibilities of designing of theory of linear noncommutative converters of random processes.

Different types of expansions contain general/common spectral [12]:

$$\xi(t) = \int_{\Lambda} \psi(t, \lambda) Z(d\lambda). \tag{1}$$

Here $\{\psi(t, \cdot)\}$ - defined family of functions of variable $\lambda \in \Lambda$, depending on time $t \in T$ as on parameter; Λ - specific space with certain algebra of sets σ; Z(·) - random measure on set Λ.

$$EZ(\Lambda_1)\overline{Z(\Lambda_2)} = F(\Lambda_1, \Lambda_2) \quad (\Lambda_1, \Lambda_2 \subset \Lambda), \tag{2}$$

where F(·, ·) - additive function of both positively determined type arguments [8]. Then from formulas (1) and (2) follows the expansion of covariance [1]:

$$r_{\xi}(t, s) = \int_{\Lambda \times \Lambda} \psi(t, \lambda) \overline{\psi(s, \mu)} F(d\lambda, d\mu) \tag{3}$$

and vice versa.

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Since function F(·, ·) characterizes the correlation of separate components of expansion (1), deserves attention the case, when these components are not correlated, i.e., when measure Z(·) is orthogonal. We have

$$EZ(\Delta_1)\overline{Z(\Delta_2)} = F(\Lambda_1 \cap \Lambda_2). \tag{4}$$

Then the spectrum of process is concentrated on the diagonal of space $\Lambda \times \Lambda$ and covariance can be presented thus:

$$r_{\xi}(t, s) = \int_{\Lambda} \psi(t, \lambda) \overline{\psi(s, \lambda)} F(d\lambda). \tag{5}$$

Expansions of such type are called canonical [9, 10]. As a result of the linear transformation of the superposition of functions $\{\psi(\cdot,\lambda)\}_{\lambda\in\Lambda}$ by operator, functioning on variable t, is obtained the same superposition of the functions

$$\Phi_A(t,\lambda) = A_t\psi(\cdot,\lambda), \tag{6}$$

called the Zade characteristics of operator A [14]. Therefore it is natural to isolate the expansions, in which instead of the arbitrary functions $\psi(\cdot, \cdot)$ stand the eigenfunctions $\varphi(\cdot, \cdot)$ of the specific operators, i.e., when

$$\Phi(t,\lambda) = \Psi(\lambda)\varphi(t,\lambda),$$

where $\Psi(\lambda)$ - the generalized frequency characteristic.

Let us name such expansions orthogonal, if they are canonical and $\{\varphi(\cdot,\lambda)\}$ - orthonormal set of eigenfunctions of assigned operator. Then set Λ can be supplied in the one-to-one correspondence with the set of its eigenvalues. Here are involved the expansions of Karunen - Loeve [8] on the segment and for Hilbert processes [13], and also representation of the T-variant random processes (TVSP) [3], for wnich there are such operators of the generalized shift/shear (OOS) T, that the covariance (with the assigned operator T) is determined by the function of one variable:

$$r_\xi(t,s) = \widetilde{T}^s R_\xi(t) \qquad (t, s \in R),$$

where $R_\xi(\cdot)$ - function, positively determined relative to OOS T; ~ - symbol of coupling. Function $R_\xi(\cdot)$ allows the expansion

$$R_{\xi}(t) = \int_{\Lambda} \varphi(t, \lambda) S(d\lambda), \tag{7}$$

where S(·) - σ-finite function on Λ. Hence follows the expansion of covariance of the type (5).

For development of commutative theory essential is the fact that for each class of variance SP it is possible to determine appropriate class of TV operators, which commutate with data of OOS $(A \smile T)$. The time characteristics of TV operators

$$g(t, s) = \widetilde{T}^s G(t), \tag{8}$$

where G(t) - pulse response, $G(t) = A\delta(t)$, and frequency - $\Psi(\lambda) = \frac{A\varphi(\cdot, \lambda)}{\varphi(t, \lambda)}$ (isomorphous response) are connected with conversion according to the eigenfunctions

$$\Psi(\lambda) = \mathfrak{K} G(t) = \int_{R} \overline{\varphi(t, \lambda)} G(t)\, dt. \tag{9}$$

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For analysis of conversions with the help of TV operators are used in time domain the methods of theory of C*-algebras with multiplication in the form of T-convolution

$$(f * g)(t) = \int_{R} f(t) \widetilde{T}^s g(s)\, ds,$$

and in generalized frequency - GELFAND isomorphism of this algebra [9]. To consecutive application operator $A = A_2 \cdot A_1$ corresponds the expression

$$\Psi(\lambda) = \Psi_{A_1}(\lambda) \Psi_{A_2}(\lambda) = \mathfrak{K}(G_{A_1} * G_{A_2}). \tag{10}$$

Theorem 1. With assigned OOS T the class of TVSP is invariant relative to TV conversions.

Proof directly follows from relationships

$$\eta(t) = A\xi(t) = (G_A * \xi)(t) = \int_\Lambda \varphi(t, \lambda) \Psi_A(\lambda) Z(d\lambda); \tag{11}$$

$$r_\eta(t, s) = \widetilde{T}^s (G_1 * \overline{G}_A * R_\xi)(t) = \int_\Lambda \varphi(t, \lambda) \overline{\varphi(s, \lambda)} | \Psi_A(\lambda) |^2 F_\xi(d\lambda). \tag{12}$$

Transference of these representations to generalized random processes is described in work [4].

During the study of noncommutative converters $(A \psi B)$ for describing their operators, besides Zade characteristics (6), we insert expansions of these characteristics relative to selected coordinate system:

$$\Phi(t, \lambda) = \int_\Lambda \psi(t, \mu) d_\mu C(\mu, \lambda). \tag{13}$$

In particular, if A - identical operator, then this formula gives the expansion of coordinate function TVSP.

Time characteristics of series connection of noncommutative converters is given by Volterra composition (see [11]):

$$g_{BA}(t, s) = (g_B \wedge g_A)(t, s) = \int_{-\infty}^{\infty} g_B(t, u) g_A(u, s) du.$$

Let us determine Δ-composition by the relationship

$$(C_1 \Delta C_2)(\nu, \lambda) = \int_\Lambda d_\nu C_1(\nu, \mu) d_\mu C_2(\mu, \lambda).$$

If $C_B(\cdot,\cdot)$, $C_A(\cdot,\cdot)$ and $C_{BA}(\cdot,\cdot)$ - expansions of Zade characteristics of corresponding operators relative to one coordinate system, then

$$d_\nu C_{BA}(\nu,\lambda)=(C_B\Delta C_A)(\nu,\lambda).$$

Let us note that when there are derivatives $c(\mu\cdot)=\frac{d}{d\mu}C(\mu,\cdot)$, Δ-composition passes into Volterra composition:

$$c_{BA}(\nu,\lambda)=\frac{d}{d\nu}C_{BA}(\nu,\lambda)=(c_B\wedge c_A)(\nu,\lambda).$$

If in formula (13) $\psi(\cdot,\ \cdot)$ the eigenfunction of operator A, then

$$C_A(\mu,\lambda)=\Psi_A(\lambda)U(\mu-\lambda),$$

where U(·) - the unit function of Heaviside.

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It is easy to demonstrate on the basis of formula (6) that under the conditions, when there are Zade characteristics of operators in question, occurs this theorem.

Theorem 2. The linear transformation of random process does not derive/conclude it from the class of those canonically represented. For the Hilbert SP it is necessary that $\Phi(t,\cdot)\in L_F^2(R)$.

Determination. Let us name T-represented SP, for which occurs the expansion of the type (1), where $\psi(\cdot,\ \cdot)$ eigenfunctions OOS, T, and measure Z(·) is nonorthogonal. Then the following confirmations are valid.

Theorem 3. If coordinate function SP allows representation (11), i.e.

$$\psi(t, \lambda) = \int_{\Lambda} \varphi(t, \mu)\, d_{\mu} C(\mu, \lambda),$$

where $\varphi(\cdot, \lambda)$ - eigenfunctions OOS T, then SP is T-representable.

Theorem 4. The class of T-representable SP is invariant relative to TV conversions.

First confirmation is obtained directly from formula (1):

$$\xi(t) = \int_{\Lambda} \varphi(t, \mu)\, dY(\mu).$$

Then two-frequency spectral function (cf [3]) will be

$$F_{Y}(\lambda, \mu) = \int_{\Lambda} C(\nu, \lambda)\, \overline{C(\mu, \nu)}\, F_{\xi}(d\nu).$$

Second confirmation ensues from relationship of type (11) when $A \omega T$, from which in this case follows

$$r_{\eta}(t, s) = \int_{\Lambda} \varphi(t, \lambda)\, \overline{\varphi(s, \mu)}\, \Psi_{A}(\lambda)\, \overline{\Psi_{A}(\mu)}\, F(d\lambda, d\mu).$$

It is evident from the last formula that the theorem given below occurs.

Theorem 5. If T-representable SP possesses the two-frequency spectral density

$$f(\lambda, \mu) = \frac{\partial^2 F(\lambda, \mu)}{\partial\lambda \partial\mu},$$

then covariance of SP is its double $\mathfrak{K}$- conversion

$$r_{\xi}(t, s) = \mathfrak{K}^2 f(\lambda, \mu),$$

and vice versa, $\mathfrak{K}$-conversion from any function of the positively determined type is covariance of T-represented SP with spectral density. Since T-representability of SP is the generalization of the concept of harmonizability, then theorems 3 and 4 lead to the following (cf. [5]).

Corollary 1. For harmonizability of Hilbert SP it is necessary and sufficient that with all values $t \in R$ its coordinate function would be $\psi(t, \cdot) \in L^2_F(\Lambda)$ and it allowed representation in the form of integral of Fourier - Stieltjes.

Corollary 2. The class of the harmonizable processes is invariant relative to invariant (in time) converters.

Example I. Let us consider simple amplitude-modulated harmonic oscillation [12]:

$$\eta(t) = \xi(t) \cos \lambda_0 t,$$

where $\xi(\cdot)$ - stationary random process.

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Since covariance SP

$$r_\eta(t, s) = R_\xi(t - s) \cos \lambda_0 t \cos \lambda_0 s,$$

its two-frequency spectral function will be

$$F_\eta(d\lambda, d\mu) = \frac{1}{4} \{[dF_\xi(\lambda + \lambda_0) + dF_\xi(\lambda - \lambda_0)] \delta(\lambda - \mu) + \\ + dF_\xi(\lambda + \lambda_0) \delta(\lambda - \mu + 2\lambda_0) + dF_\xi(\lambda - \lambda_0) \delta(\lambda - \mu - 2\lambda_0)\} d\mu,$$

i.e., SP itself $\eta(\cdot)$ is harmonizable; its spectrum is concentrated on diagonal $\mu=\lambda$ and straight lines $\mu=\lambda+2\lambda_0$ and $\mu=\lambda-2\lambda_0$ parallel to it. This process is a special case of periodically correlated SP [2], for which

$$r(t+T,\ s+T)=r(t,\ s)\qquad (\forall t,\ s;\ T=\mathrm{const}).$$

Hence for its transform with the help of the invariant operator it is easy to deduce

$$r_1(t+\tau,\ t)=\sum_k e^{ik\frac{2\pi}{T}t}F_k(\tau),$$

where

$$F_k(\tau)=\int_{-\infty}^{\infty}e^{i\lambda\tau}\Psi_A(\lambda)\overline{\Psi_A\left(\lambda-\frac{2\pi}{T}k\right)}dG_k(\lambda). \qquad (14)$$

Function $G_k(\cdot)$ is connected with the two-frequency spectral function of the converted process with the relationship

$$F(d\lambda,\ d\mu)=\sum_k\delta\left(\lambda-\mu-\frac{2\pi}{T}k\right)dG_k(\lambda)\,d\mu.$$

Thus, converted SP is also periodically correlated when integral (14) exists. Hence it follows that the amplitude-modulated oscillation/vibration remains the same, also, after conversions with the help of the systems with the constant parameters.

Example II. As an example of expansions with the orthogonal functions let us consider sampling theorems for TVSP [3] in the case of OOS, generated by the equation of Sturm-Liouville (in the normal form):

$$\xi(t)=\sum_k a_A(t,\ t_k)\xi(t_k). \qquad (15)$$

where $a_\Lambda(t, s)$ - cardinal function,

$$a_\Lambda(t, s) = \sqrt{\beta_\Lambda(t)\beta_\Lambda(s)}\, g_\Lambda(t, s);$$

$\beta_\Lambda(\cdot)$ - renormalization factor; $g_\Lambda(\cdot, \cdot)$ - pulse response of the generalized low-pass filter with band $[0, \Lambda]$. Functions $a_\Lambda(\cdot, \cdot)$ are orthogonal in the sense of the equality

$$\int\limits_R a_\Lambda(t, t_k)\overline{a_\Lambda(t, t_l)}\, dt = \delta_{kl}$$

Covariance of process (15) takes form

$$r_\xi(t, s) = \sum_{k,l} a_\Lambda(t, t_k)\overline{a_\Lambda(s, t_l)}\, r_\xi(t_k, t_l).$$

Example III. Let us note the case of the existence of relationships of the type (13). If functions $\varphi(\cdot, \cdot)$ and $\psi(\cdot, \cdot)$ are the eigenfunctions of the operators of Sturm - Liouville, which correspond to eigenvalues of t, then under specified boundary conditions [6] there is a kernel $K(\Lambda, \mu)$, with which in formula (13) we have

$$c(\mu, \lambda) = \frac{d}{d\mu} C(\mu, \lambda) = \chi_{[0,\lambda]}(\mu) K(\lambda, \mu) + \delta(\lambda - \mu),$$

where $\chi_A(\cdot)$ - indicator of set A.

Examples of use of such relationships are in work [5], and also in theory of orthogonal filters [6].

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ON PERIODICALLY CORRELATED RANDOM PROCESSES AND CONVERTERS WITH PERIODICALLY CHANGING PARAMETERS.

Ya. P. Dragan, (L'vov).

Possible types of periodically correlated random processes are examined and is established their connection with systems, which possess periodically changing parameters, and also with harmonizable, periodic and possessing spectra processes. No tables. Bibliog. 15.

Many works are devoted to the study of the properties of the periodically correlated (PK) random processes [2, 3, 4, 5, 8]. Purpose of this article - to investigate possible types of periodically correlated random processes [PKSP] and to establish their connection with the systems, which possess periodically changing in time parameters (PIP).

Definition 1. Let us name τ-PKSP such process, the covariance of pulsations of which with all values $t \in R$ satisfies the condition

$$r(s+\tau, t+\tau) = r(s, t), \tag{1}$$

where $\tau \in R$ - fixed number.

PKSP is convenient to describe by function

$$b(t, u) = r(t+u, t). \tag{2}$$

Then it follows from condition (1) that function b(·, u) is

τ-periodic. Let us assume that with all values $u \in R$ we have $b(\cdot, u) \in L(0, \tau)$. Then occurs the representation

$$b(t, u) = \sum_{k=-\infty}^{\infty} B_k(u) e^{i\frac{2\pi}{\tau}ku}, \tag{3}$$

where

$$B_k(u) = \frac{1}{\tau} \int_0^{\tau} b(t, u) e^{-i\frac{2\pi}{\tau}kt} dt. \tag{4}$$

If τ-periodic is not only covariance of pulsations $(\cdot)_{\overset{0}{\xi}}$ of process $\xi(t) = \overset{0}{\xi}(t) + m_\xi(t)$, but also covariance of process itself, then we find from relationship

$$r_\xi(t, s) = r_\xi^0(t, s) + m_\xi(t)\overline{m_\xi(s)}$$

and equality (1) that $m_\xi(\cdot)$ is τ-periodic function.

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With satisfaction of this condition, sometimes introduced in the definition of PKSP (for example, see [3]) [1], and, furthermore, if $m_\xi(\cdot) \in L(0, \tau)$, then

$$m_\xi(t) = \sum_{k=-\infty}^{\infty} m_k e^{i\frac{2\pi}{\tau}kt},$$

where, as usual,

$$m_k = \frac{1}{\tau} \int_0^{\tau} m_\xi(t) e^{-i\frac{2\pi}{\tau}kt} dt. \tag{5}$$

FOOTNOTE [1]. Such SP they call still periodic of the second order or periodic in the broad sense [11]. ENDFOOTNOTE.

If in definition (1) $r(\cdot, \cdot)$ exists in sense of usual functions and $b(\cdot, u)$ with all values $u \in R$ is continuous, then in formulas of type (11) (see below) figure positively determined functions $V(\cdot)$ (according to Bochner). In other cases they generalized [1, 13], then measure F(·) is not limited, but exponential increase/growth. Analogously, if b(t, ·) is the generalized almost periodic function, then correspondingly is changed the determination of limits in the expressions for the average/mean value of M · [4].

After designating with arbitrary value $\theta \in R$ through

$$\mu_k(\theta) = \frac{1}{\theta}\int_0^{\theta} \xi(t)\, e^{-i\frac{2\pi}{\tau}kt}\, dt \tag{6}$$

coefficients of root-mean-square (SK) Fourier series of pulsations PKSP ξ(·) in segment $(0, \theta)$, we investigate ergodic properties of this process. From formula (6) we find

$$E\,|\mu_k(\theta)|^2 = \frac{1}{\theta^2}\int_0^{\theta}\int_0^{\theta} r(s, t)\, e^{-ik\frac{2\pi}{\tau}(t-s)}\, dt\, ds = \frac{1}{\theta^2}\int_0^{\theta} dt \int_{t-\theta}^{t} b(t, u)\, e^{-i\frac{2\pi}{\tau}ku}\, du.$$

Let us assume that with all values $t \in R$ function b(t, ·) is uniform almost periodic [4]. For all values $\lambda \in R$ evenly from parameter $a \in R$ there is a limit

$$\beta(t_0\, \lambda) = M_u\{b(t, u)\, e^{-iu\lambda}\} = \lim_{\theta\to\infty} \frac{1}{\theta}\int_a^{\theta+a} b(t, u)\, e^{-i\lambda u}\, du$$

different from zero on denumerable set $\{\lambda_k\}_{k,\overline{1\infty}}$ - the set of indices of Fourier function b(t, ·), equal, obviously, to the union of the sets of indices of Fourier function $B_k(\cdot)$ [4]. Then on the basis of the property of the integral of the sequence, which possesses the summarized majorant,

$$\lim_{\theta\to\infty} |\mu_k(\theta)|^2 = M\,|\beta(\cdot, \lambda)|.$$

Hence follows the validity of this theorem.

Theorem 1. If one of the conditions is implemented:

1) $b(t, \cdot) \in L(R)$; 2) $b(t, \cdot)$ - almost periodic function, Fourier's indices of which are different from numbers $\left\{k\frac{2\pi}{\tau}\right\}_{k=\overline{1,\infty}}$; 3) $B_0(u) \to 0$, then

$$\lim_{\theta\to\infty} |\mu_k(\theta)|^2 = 0,$$

i.e., in the sense of SK of the limit

$$m_k = M\{\xi(t)e^{-i\frac{2\pi}{\tau}kt}\}.$$

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This theorem characterizes conditions, with which mathematical expectation of PKSP can be calculated according to one realization [3].

Definition 2. Let us name SP τ-periodic, if the covariance of its pulsations with all values $t \in R$ satisfies the condition

$$r(s+k\tau, t+l\tau) = r(s, t) \quad (\tau \in R), \tag{7}$$

where k, sl - whole numbers.

From formula (7) it follows that function b(.,.) is periodic on both arguments, and

$$b(t, u) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} B_{kl} e^{i\frac{2\pi}{\tau}(kt+lu)} = \sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} B_{k,k-n} e^{i\frac{2\pi}{\tau}(k(t+u)-nu)}, \tag{8}$$

whence it follows that τ-periodic process can be represented in the form

$$\xi(t) = \sum_{k=-\infty}^{\infty} \xi_k e^{i\frac{2\pi}{\tau}kt}.$$

In most general case function b(t, rd) is represented by integral of Fourier - Stieltjes. Then from expansion (3) it follows that this representation possesses each function $B_k(\cdot)$ $(k = \overline{-\infty, \infty})$:

Hence we find

$$B_k(u) = \int_{-\infty}^{\infty} e^{it\lambda} dF_k(\lambda).$$

$$b(t, u) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(u\lambda - t\mu)} d_\lambda d_\mu F(\lambda, \mu),$$

where

$$F(\lambda, \mu) = \sum_{k=-\infty}^{\infty} F_k(\lambda) U\left(\mu + k\frac{2\pi}{\tau}\right);$$

U(·) - Heaviside's function.

Then covariance takes form

$$r(s, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i(s\lambda - t\nu)} d_\lambda d_\nu S(\lambda, \nu).$$

Thus, it is possible to formulate theorem.

Theorem 2. Harmonizable SP will be τ-periodically correlated only in such a case, when its two-frequency function will be represented in the form

$$S(\lambda, \nu) = F(\lambda, \nu - \lambda) = \sum_{k=-\infty}^{\infty} \left\{F_k(\lambda) U\left(\nu - \lambda + k\frac{2\pi}{\tau}\right) + F_k(\nu) U\left(\lambda - \nu + k\frac{2\pi}{\tau}\right)\right\}.$$

Let us point out two special cases

1. $B_k(\cdot) \in L(R)$ $(k \ \overline{-\infty, \infty})$. Taking into account that according to the theory of generalized functions [13] $\frac{dI(\lambda)}{d\lambda} = \delta(\lambda)$, where $\lambda(\cdot)$ - Dirac's measure, we obtain expression for the spectral density

$$s(\lambda, \nu) = \sum_{k=-\infty}^{\infty} f_k(\lambda)\, \delta\left(\nu - \lambda + k\frac{2\pi}{\tau}\right) \; (f_k(\lambda) = F_k(\lambda)).$$

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2. $B_k(\cdot)$ τ-periodic functions. Then for τ-periodic SP [8] we have

$$s(\lambda, \nu) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} B_{kl}\, \delta\left(\nu - [k - l]\frac{2\pi}{\tau}\right) \delta\left(\lambda - l\frac{2\pi}{\tau}\right).$$

It is evident from resulting expressions that in first case spectral measure is concentrated on straight lines $\lambda - \nu = k\frac{2\pi}{\tau}$, parallel to diagonal of quardant, and in the second - at points, coordinates of which are multiple to relation $\frac{2\pi}{\tau}$.

Formulas (2) and (3) make it possible to study so-called averaged spectra of this class of transient SP [10, 12].

Definition 3. SP let us name possessing spectrum, if with all values $u \in R$ there is a function

$$B(u) = M\{b(\cdot, u)\}. \tag{9}$$

With the use of property of average [4]

$$M_t\{f(t + a)\} = M\{f(\cdot)\}$$

easily is proven the positive definiteness of function (9):

$$\sum_{i,j=1}^{N} B(u_i - u_j)\alpha_i\bar{\alpha}_j = \sum_{i,j=1}^{N} M\{b(\cdot, u_i - u_j)\}\alpha_i\bar{\alpha}_j =$$
$$= \sum_{i,j=1}^{N} M_t\{r(t + [u_i - u_j], t)\}\alpha_i\bar{\alpha}_j = M_t\left\{\sum_{i,j=1}^{N} r(t + u_i, t + u_j)\alpha_i\bar{\alpha}_j\right\} \geqslant 0. \quad (10)$$

Hence, as usual, (if function B(·) is continuous) follows the representation

$$B(u) = \int_{-\infty}^{\infty} e^{iu\lambda} F(d\lambda), \quad (11)$$

where F(·) - positive limited measure, called the spectrum of SP $\xi(\cdot)$ [10].

Where λ - Kronecker's symbol, it follows from representation (3) on the basis of property of orthogonality

$$M\{e^{i\lambda u}\} = \delta_\lambda,$$

that for PKSP function (9) exists and

$$B(u) = B_0(u). \quad (12)$$

Thus, according to condition (10) and equality (12) function $B_0(u)$ in representation (3) of covariance PKSP is positively determined (cf. [3]), i.e., it possesses property of covariance of stationary SP. Therefore it is possible to consider that it characterizes the properties of stationary approximation/approach to PKSP. Then the averaged spectrum for PKSP used in the theory of connection [12, 14] coincides with the spectrum of its stationary approximation/approach. It is obvious that according to it it is not possible to determine completely the correlation properties SP (cf.

[14]).

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On the basis of the fact that class of covariances coincides with class of positively determined type functions, for which

$$\sum_{i,j=1}^{N} r(t_i, t_j)\alpha_i\bar{\alpha}_j \geqslant 0,$$

taking into account relationship/ratio (cf. formula (4))

$$B_k(u) = M_t\{b(t, u)e^{-ik\frac{2\pi}{\tau}t}\},$$

easily is derived/concluded condition

$$\sum_{i,j=1}^{N} C_i\bar{C}_j B_{k_i-k_j}(t_i - t_j)e^{i(k_i-k_j)\frac{2\pi}{\tau}t_j} = M_s\left\{E\left|\sum_{j=1}^{N} C_j\xi(s+t_j)e^{-ik_j\frac{2\pi}{\tau}s}\right|^2\right\} \geqslant 0..$$

After taking into consideration the property of functions $B_k(\cdot)$

$$B_k(u) = \overline{B_{-k}(u)}\,e^{ik\frac{2\pi}{\tau}u}$$

and after assuming $k_i = k\delta_i^0$, from the last equality let us find the characteristic relationship

$$\sum_{j=1}^{N} \{C_j B_k(t_j)e^{ik\frac{2\pi}{\tau}t_j} + \bar{C}_j\overline{B_k(t_j)}\} \geqslant 0,$$

from which for function $B_k(\cdot)$ follows the representation

$$B_k(s-t) = e^{-i\frac{2\pi}{\tau}kt}\sum_{k=-\infty}^{\infty} e^{i\frac{2\pi}{\tau}n(s-t)}\varrho_k(s-t),\ \varrho_k(u) = \int_{-\infty}^{\infty} e^{iu\lambda}\Phi_k(d\lambda) \qquad (13)$$

where $\Phi_k(\cdot)$ - limited measure.

For research of linear transformations of PKSP let us introduce appropriate concepts. Following Friedrichs [7], for the assigned function $\varphi(\cdot)$ let us determine operator (where $p = \frac{d}{dt}$) the multiplication of Fourier transform of certain function by the function $\varphi(\lambda)$. Then function $a(t, \mu)$ we name the symbol of operator $A=a(t, p)$:

$$Au(t) = \mathfrak{f}^{-1}\{a(t, \cdot)\mathfrak{f}u\},$$

where $\mathfrak{f}$ the symbol of Fourier transform. The concept of the symbol of operator generalizes the concept of the transient function of system with parameters [6] changing in time. A special case of the degenerate operator, whose symbol takes the form

$$a(t, \mu) = \sum_j a_j(t) P_j(\mu).$$

is differential operator with the variable coefficients ($P_j(\cdot)$ - polynomial of the j degree) (cf. [9]).

Definition 4. τ-periodic let us name the operator, whose symbol $a(\cdot, \mu)$ with all values $\mu \in R$ is τ-periodic function.

Theorem 3. τ-periodic operator can be represented in the form of the number on the invariant operators, coefficients of which are the harmonic modulators:

$$A = \sum_{k=-\infty}^{\infty} A_k(p) e^{i\frac{2\pi}{\tau}kt}, \tag{14}$$

which ensues from expansion into Fourier series of the symbol of operator. Let us consider transform $\eta(\cdot)$ of stationary SP $\xi(\cdot)$ with the system PIP, described by τ-periodic operator [13]:

$$\eta(t) = A\xi(t) = \sum_{k=-\infty}^{\infty} \xi_k(t) e^{i\frac{2\pi}{\tau}kt},$$

where

$$\xi_k(t) = A_k(p)\,\xi(t) = \int_{-\infty}^{\infty} \psi_k(\lambda)\, e^{it\lambda} Z_\xi(d\lambda) \quad (\psi_k(\lambda) = A_k(i\lambda)).$$

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Its covariance is represented by expression

$$r_\eta(s, t) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} R_{kl}(s-t)\, e^{i\frac{2\pi}{\tau}(ks-lt)},$$

where are introduced designations

$$R_{kl}(u) = \overline{R_{lk}(u)} = E\xi_k(t+u)\,\overline{\xi_l(t)}.$$

Hence it is apparent that $\eta(\cdot)$ is τ-PKSP, moreover in appropriate expansion of type (3) of its covariance function

$$B_k(u) = \sum_{n=-\infty}^{\infty} R_{n,n-k}(u)\, e^{i\frac{2\pi}{\tau}nu} = \int_{-\infty}^{\infty} e^{i\lambda u} \Psi_k(u, \lambda)\, dS_\xi(\lambda),$$

where

$$\Psi_k(u, \lambda) = \sum_{n=-\infty}^{\infty} \psi_n(\lambda)\, \psi_{n-k}(\lambda)\, e^{i\frac{2\pi}{\tau}nu},$$

it is τ-periodic on u. The convergence of last number follows from convergence of series (13):

$$|\Psi_k(u, \lambda)| \leqslant \sum_{n=-\infty}^{\infty} |\psi_n(\lambda)|\, |\psi_{n-k}(\lambda)| < \left(\sum_{n=-\infty}^{\infty} |\psi_n(\lambda)|^2\right)^{\frac{1}{2}}.$$

On the basis of formula (12) for the stationary approach to this process we obtain

$$B(u) = \sum_{n=-\infty}^{\infty} B_{nn}(u)\, e^{i\frac{2\pi}{\tau}ku} = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |\psi_n(\lambda)|^2 e^{i\frac{2\pi}{\tau}nu} e^{iu\lambda} dS_\xi(\lambda).$$

i.e. for its existence must be satisfied condition $a(t, \cdot)\in H_{s_t}$. Let us note that for the differential operator of the n order

$$A=\sum_{l=0}^{n} a_l(t)\,p^l$$

with τ-periodic coefficients in representation (14)

$$A_k(p)=\sum_{l=0}^{n} a_{lk}p^l,$$

where a_{lk} - k Fourier function coefficient $a_l(\cdot)$. Using the fact that the symbol of product C=AB of operators [7] is the symbol

$$c(t,\mu)=\sum_k \frac{1}{k!}((-ip)^k a(t,\mu))\left(\frac{\partial}{\partial\mu}\right)^k b(t,\mu),$$

on the basis of that presented it is possible to formulate theorems.

Theorem 4. Transform of τ-periodic SP with the help of the τ-periodic operator is τ-periodic SP, while transform of stationary SP by τ-periodic operator is PKSP.

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Theorem 5. Conversion of PKSP with the help of τ-periodic operator does not derive/conclude from the class PKSP.

Invariant operator can be treated as degenerate periodic; therefore following confirmation ensues from the latter/last theorem.

Theorem 6. Conversion with the help of the invariant operator does not derive/conclude from the class PKSP. This theorem is a special case of the theorem, proved in work [5], since class of PKSP enters into the class of harmonizable SP [3].

Theorem 6 can be easily demonstrated, on the basis of representation of PKSP itself, which we obtain, proposing in representation $B_k(\cdot)$ (13), that

$$\varrho_k(u) = \sum_{n=-\infty}^{\infty} r_{n,\,k-n}(u).$$

Since this sum of positive organic measures is the positive limited measure, then, assuming

$$r_{n,k-n}(u) = \int_{-\infty}^{\infty} e^{iu\lambda} F_{n,k-n}(d\lambda) \quad \left(\sum_{n=-\infty}^{\infty} F_{n,k-n}(\Delta) = \Phi_k(\Delta)\right)$$

and considering that the mutual covariance of stationary SP

$\xi_n(\cdot)$ и $\xi_{n-k}(\cdot)$ takes the form

$$r_{n,k-n}(u) = E\xi_n(t+u)\overline{\xi_{n-k}(t)} \qquad (n,\, k = \overline{-\infty, \infty}),$$

we find that PKSP always allows representation in the form

$$\xi(t) = \sum_{k=-\infty}^{\infty} e^{i\frac{2\pi}{\tau}kt} \xi_k(t),$$

where $\{\xi_k(\cdot)\}_{k=\overline{-\infty,\infty}}$ - thus the selected stationary SP. Their covariances satisfy the conditions

$$B_k(u) = \sum_{n=-\infty}^{\infty} r_{n,n-k}(u) e^{i\frac{2\pi}{\tau}nu}.$$

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